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DISCUSSION OF LAMINAR TO TURBULENT FLOW IN A WIDE OPEN CHANNEL PROCEEDINGS-SEPARATE NO. 188

Yuichi Iwagaki.[13]—In the hydraulic laboratory of Kyoto University (Kyoto, Japan), the writer has performed an experiment[14] similar to that conducted by the author. In this experiment, a rectangular section, planed wood flume 40 cm (1.31 ft) wide, 19 cm (0.62 ft) deep, and approximately 18 m (59.1 ft) long (effective length of 10 m (32.8 ft)) was used; the experimental data were taken for slopes of 0.0021, 0.005, 0.01, 0.02, and 0.024, and the water depth varied from 0.16 cm (0.0053 ft) to 3.85 cm (0.126 ft).

Mr. Owen treated his experimental results as two-dimensional flow. Taking the maximum water depth as 0.3 ft, the ratio of water depth y to hydraulic radius R becomes

$$\frac{y}{R}=1+\frac{2\,y}{B}=1+\frac{0.6}{1.5}=1.4 \qquad (12)$$

in which B is the width of the flume. The flow of water having such a depth cannot be regarded as two-dimensional flow. The relationship obtained by the

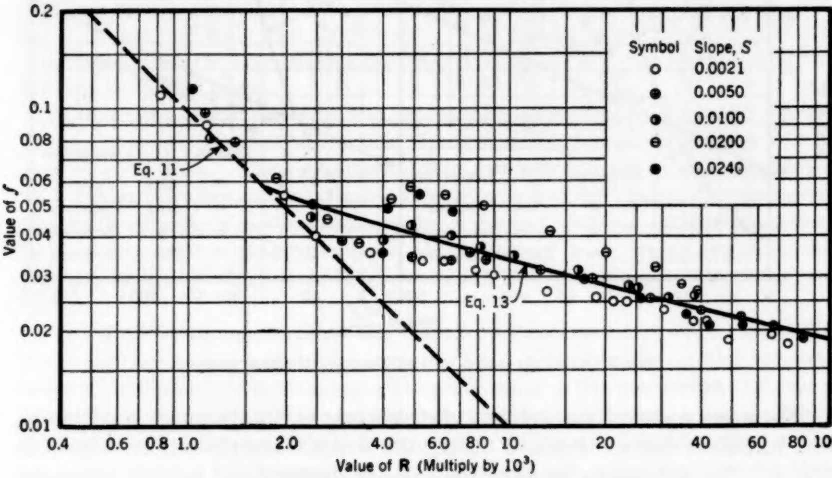

Fig. 4.—Relation Between the Friction Factor and the Reynolds Number

writer between the friction factor f and the Reynolds number $\mathbf{R}$ is shown in Fig. 4. The solid line in Fig. 4 satisfies

$$\frac{1}{\sqrt{f}} = 2.033 \log (\mathbf{R}\sqrt{f}) - 1.081 \qquad (13)$$

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[14] "Studies on the Thin Sheet Flow (1st. Report)," by Tojiro Ishihara, Yuichi Iwagaki, and Takeshi Goda, *Transactions*, Japan Soc. of Civ. Engrs., No. 6, 1951, pp. 31–38 (in Japanese).

Eq. 13 can also be obtained from the equation for the distribution of velocity in a smooth pipe

$$\frac{u}{V_f} = 5.5 + 5.75 \log \frac{V_f Z}{\nu} \qquad (14)$$

in which u is the velocity at the distance Z from the bottom and V_f the friction velocity. It is seen in Fig. 4 that the writer's experimental results tend to be plotted below the curve of Eq. 13 in the case of gentle slopes, and above the curve in the case of steep slopes. The reasons for these results have been explained by the writer.[15] The author's experimental data in the turbulent region (Fig. 1) show an almost constant value for the friction factor. This can be explained by the fact that the results obtained three dimensionally in Mr. Owen's experiment were expressed two dimensionally.

The values of the Reynolds number at the change from laminar to transitional flow and from transitional to turbulent flow are found in Fig. 4 to be 2,000 and 5,000, respectively. Both of these critical values are smaller than those found by the author, and this difference is caused by the amount of initial disturbance resulting from the shape of the entrance to the flume.

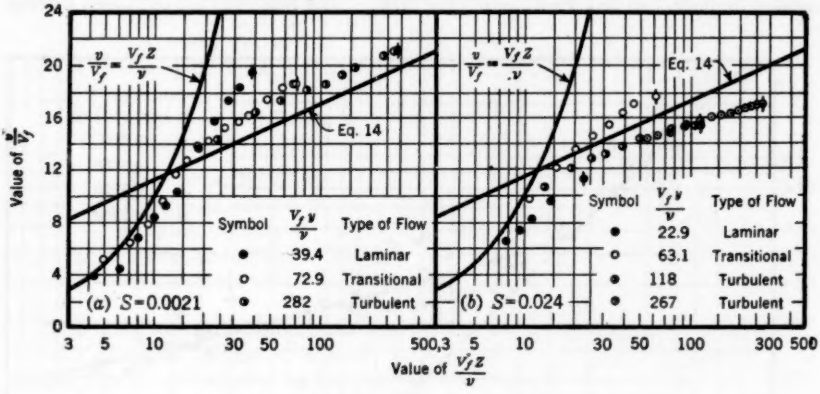

FIG. 5.—SOME EXAMPLES OF THE VELOCITY DISTRIBUTIONS

Some examples of the velocity distributions in the laminar, transitional, and turbulent regions obtained during the writer's experiments are shown in Fig. 5. The differences between the velocity distributions in each region are also shown in Fig. 5.

Investigations of flow with low Reynolds numbers are necessary for the examination of the problems of drainage and soil erosion. It is also important to develop the relationship between the type of flow, either laminar or turbulent, and sediment transportation.

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[15] "On the Laws of Resistance to Turbulent Flow in Open Smooth Channels," by Yuichi Iwagaki, *Memoirs*, Faculty of Eng., Kyoto Univ., Vol. 15, No. 1, January, 1953, pp. 27–40.

J. W. Delleur,[16] J. M. ASCE.—The determination of the Reynolds number at which the transition from laminar flow to turbulent flow occurs in open-channel flow, and the relationship between the friction factor and the Reynolds number, fill a void in the knowledge of open-channel flow. The departure of the experimental data from the Blasius equation, shown in Fig. 1, is of special interest to those engineers investigating turbulent flow in open channels.

The applications of Mr. Owen's results are limited to smooth rectangular channels as his investigations were performed in a glass-walled flume with a smooth brass bottom. Determination of the effect of roughness, and preparation of a system of curves, similar to that shown in Fig. 1 (in which the relative roughness is a parameter), would seem to be the next steps to be undertaken.

The points which seem to need some clarification are whether the author's experiments actually led to a condition of uniform flow, and whether the flow was sufficiently two dimensional to justify the neglect of the side-wall effects.

First it seems questionable that a depth of 0.3 ft in a 1.5-ft-wide channel (that is, a ratio of width to depth of $\frac{1}{5}$) can be considered to approximate the flow conditions that exist in a channel of infinite width. The writer has studied phenomena in open-channel flow connected with the development of the turbulent boundary layer along the bottom of an open horizontal channel. The depth-to-width ratio which was used varied from $\frac{1}{6}$ to $\frac{1}{8}$, and it was found that the flow was not truly two dimensional. The results obtained show that the side-wall boundary layers have a substantial effect on the rate of growth of the bottom boundary layer.

The second point concerns the assumption of uniform flow made by the author in all his theoretical considerations and also his interpretation of the experimental data. The experimental flow was gaged to assure uniform depth. This gaging provided a constant mean velocity, but it did not necessarily assure a uniform velocity distribution throughout the test section. A uniform velocity distribution would be obtained if the boundary layer, which began at the entrance of the channel, were fully developed at the upstream end of the test section.

In Mr. Owen's experiments the boundary layer may not have been fully developed before the test section was reached. The formula for the development of the boundary layer along a flat plate leads to the conclusion that, for a Reynolds number of 3,000, under the test conditions indicated, a length of 9 ft (almost half the channel length) is required to obtain a full development of the boundary layer. Furthermore, A. E. Craya and the writer have found that the development of the boundary layer along the bottom of an horizontal channel is slower than that along a flat plate in an infinite fluid.[17] In unconfined flow, as along a flat plate in an infinite fluid, the displacement of the outer flow caused by the boundary layer disturbs the original flow around the body only slightly. However, in open-channel flow the boundary layer has a sub-

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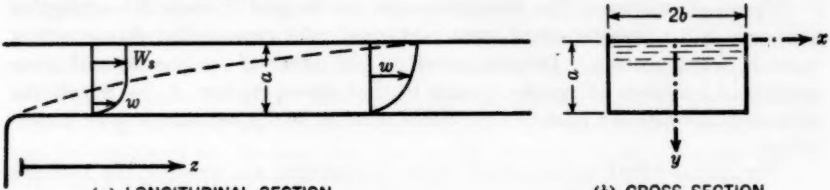
[16] Instructor in Civ. Eng., Columbia Univ., New York, N. Y.

[17] "An Analysis of Boundary Layer Growth in Open Conduits near Critical Regime," by A. E. Craya and J. W. Delleur, *CU-1-52-ONR-266 (10)-CE*, Dept. of Civ. Eng., Columbia Univ., New York, N. Y., 1952 (report to the Office of Naval Research).

stantial influence on the frictionless flow outside the boundary layer. The effect of the side walls is to decrease further the rate of growth of the boundary layer along the bottom of the channel. It then appears that, in some of the author's experiments, the boundary layer may not have been completely developed before it reached the test section of the channel—that is, between the two electric point gages.



(a) LONGITUDINAL SECTION (b) CROSS SECTION

FIG. 6.—SECTIONS OF THE EXPERIMENTAL CHANNEL

To illustrate the boundary-layer growth, Fig. 6 shows the sections of the channel used by the writer to obtain the experimental data shown in Fig. 7.

Velocity-distribution measurements were taken for several flow conditions—ranging from laminar flow to turbulent flow—and for different values of the ratio, z/a. The channel used (Fig. 6) was 13.5 in. wide and 15 ft long. The

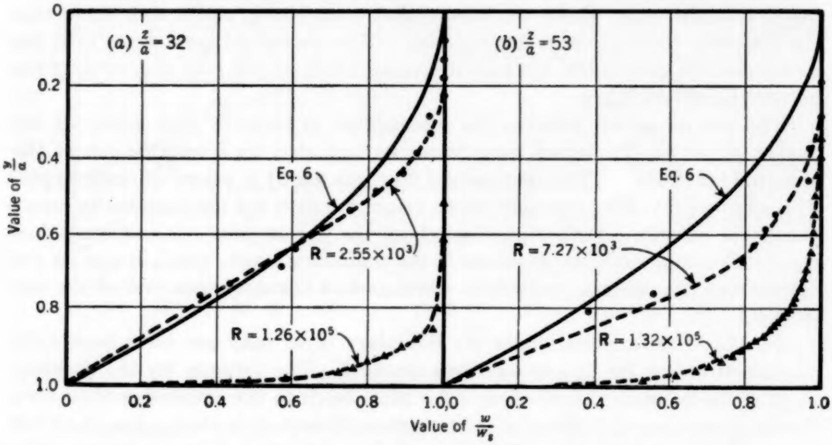


FIG. 7.—VELOCITY PROFILES FOR LAMINAR AND TURBULENT FLOWS AT THE SAME RELATIVE ABSCISSA

velocity measurements were taken by using a pitot tube made of a hypodermic needle (outside diameter of 0.060 in. or 0.027 in.) mounted on a streamlined lucite stem. The vertical motion of the pitot tube was controlled to the nearest 0.0005 in. by a traverse gear. The velocity head was measured to the nearest 0.001 in. by a micromanometer which was described by R. C. Pankhurst[18] and by E. Ower.[19]

[18] "Wind Tunnel Technique," by R. C. Pankhurst and D. W. Holder, Sir Isaac Pitman & Sons, Ltd., London, England, 1952.

[19] "The Measurement of Air Flow," by E. Ower, Chapman & Hall, London, England, 1933.

For the purpose of comparison, the theoretical velocity distribution given by Eq. 6 has been plotted in Fig. 7. It can be seen that the velocity distribution given by Eq. 3, at least for $x = 0$, converges rapidly to the values obtained from Eq. 6 when the ratio b/a in Eq. 3 increases beyond 10.

In Fig. 7(*a*), for $z/a = 32$, the curve for $\mathsf{R} = 2.55 \times 10^3$ shows that for this laminar flow the boundary layer is not fully developed and reaches only 75% of the depth of flow. However, there is good agreement between the experimental and the theoretical curves, and the values of the friction factor will not be greatly affected. In terms of a depth of 0.3 ft, the ratio z/a corresponds to an abscissa —$z = 9.6$ ft— almost at the center of the channel and within the test section. The curve (Fig. 7(*a*)) for turbulent flow at the same value of z/a but for $\mathsf{R} = 1.26 \times 10^5$ shows that the boundary layer is much less developed, and that it reaches approximately 40% of the depth of flow. This fact throws a greater doubt on the uniformity of flow for the turbulent range of Mr. Owen's experiments, and on the applicability of the friction factors computed therefrom to uniform flow.

In Fig. 7(*b*) are presented data similar to those shown in Fig. 7(*a*) for $z/a = 53$, and for a flow in the transitional zone with $\mathsf{R} = 7.27 \times 10^3$ as well as for a turbulent flow with $\mathsf{R} = 1.32 \times 10^5$. All curves for turbulent flow in Fig. 7 were taken essentially for the same flow conditions. The growth of the boundary layer was from 40% to 65% of the depth of flow (Fig. 7(*a*) to Fig. 7(*b*)).

The actual depths a and the surface velocities W_s used by the writer are given in Table 1.

TABLE 1.—Depths and Surface Velocities Corresponding to Fig. 7

Relative abscissa z/a	Reynolds number R	Depth a, in feet	Surface velocity W_s, in feet per second
32	2.55×10^3	0.0410	0.150
32	1.26×10^5	0.1715	1.660
53	7.27×10^3	0.0965	0.182
53	1.32×10^5	0.1643	1.800

Regarding the velocity measurements in turbulent flow, it can be added that a roughness element was placed at the entrance of the channel so as to insure a turbulent boundary layer beginning at that point.

It would seem that Mr. Owen's experiments for laminar flow achieved a velocity distribution close to the theoretical distribution. However, in the case of turbulent flow, the experiments probably differed substantially from the assumed uniform flow. To assure uniform flow in the test section a longer entrance zone is probably necessary. For a closer analysis of data taken near the entrance, the boundary layer and side-wall effect should be considered.

Ralph W. Powell,[20] and Chesley J. Posey,[21] Members, ASCE.—The experimental results recorded by Mr. Owen are interesting and valuable. Eq. 3, however, is an equation which does not converge when $b > a$.

The difficulty is caused by the fact that Mr. Timoshenko measured a in the x-direction and b in the y-direction, and the author has interchanged them. Mr. Owen has tried to correct for this by interchanging a and b in Mr. Timoshenko's equation; but if b is to approach infinity, it must remain larger than a; thus, x and y should have been interchanged, and Eq. 3 should have been written as

$$w = \frac{16\,K\,a^2}{\pi^3} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n^3} (-1)^{(n-1)/2} \left[1 - \frac{\cosh \frac{n\,\pi\,x}{2\,a}}{\cosh \frac{n\,\pi\,b}{2\,a}}\right] \cos \frac{n\,\pi\,y}{2\,a} \ldots (15)$$

which reduces to

$$Q = \frac{2}{3}\,K\,a^3\,b \left(1 - \frac{192\,a}{\pi^5\,b} \sum_{n=1,3,5,\cdots}^{\infty} \frac{1}{n^5} \tanh \frac{\pi\,n\,b}{2\,a}\right) \ldots\ldots\ldots (16)$$

rather than Eq. 5. When $b = \infty$, Eq. 16 results in

$$q = \frac{Q}{2\,b} = \frac{K\,a^3}{3} = \frac{g\,S\,a^3}{3\,\nu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

which is the equation for an infinitely wide channel.

The maximum recorded depths in the 1.5-ft-wide channel were 0.3 ft. This causes some doubt as to whether the data should have been computed as though the width-to-depth ratio were infinite. On substituting $b/a = 5$ into Eq. 16, one finds that the laminar-flow discharge is theoretically reduced 12.6% by the effect of the side walls. The hydraulic radius in this case is 28.5% less than the depth. If the width-to-depth ratio of the various test runs had been similarly accounted for in Fig. 1, the points near the right side, where the discrepancy is the largest, would have moved downward and to the left, perhaps enough to give a more obvious downward trend to the curve in the turbulent range.

Although no theoretical solution is available for the turbulent case, the laminar solution (in which the actual width is taken into account) would seem to provide a sounder basis for the reduction of data than that in which the width-to-depth ratio is assumed to be infinite.

The statement in Sect. 3 that the

> "* * * velocity distribution and energy loss for an open channel of depth a and width $2\,b$ (Fig. 3) are exactly the same as those of one half of a closed conduit of depth $2\,a$ and width $2\,b$"

is open to question. The statement may be approximately true with regard to the energy loss when the flow is laminar (as shown by the author's experiments), but it is doubtful if it is true for velocity distribution, and the statement certainly is not true for either velocity distribution or energy loss when the flow is turbulent. If the statement were true, the maximum velocity would be at the

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[20] Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[21] Prof. and Head, Dept. of Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

middle of the free surface, whereas experiments show that the maximum velocity is generally depressed below the surface. Experiments have also shown that the discharge of a rectangular channel containing water flowing with uniform depth is not equal to half the discharge of a rectangular pipe twice as deep, flowing full, with the same hydraulic-gradient slope. Garbis H. Keulegan introduced the symbol $\bar{\epsilon}$ to measure the apparent "drag of the free surface" or, more accurately, the effect of the angle between the free surface and the side walls.[22] It appears that this effect is a three-dimensional effect, and that there is double-spiral flow when there is a free surface, which makes the resistance different from that in a full flowing pipe.

It is not an established fact that velocity distributions for laminar flow in open channels correspond to half of the solutions of the Navier-Stokes equations for closed conduits. Measurements made by Ellis B. Pickett, in a triangular channel with one side vertical, showed a systematic deviation from the symmetry demanded by this type of theoretical solution.[23] No explanation seemed possible except in terms of an effect of the free surface.

There has been some difference of opinion as to the lower value of the critical Reynolds number for open-channel flow, and the author's value of approximately 4,000 is a significant contribution. The paper[6] by Mr. Straub, (based on data by Edward Silberman), reported a transition range of Reynolds numbers for a rectangular channel from 2,800 to 7,000. Other experimenters[24,25,26,27,28] have found somewhat different values and have conclusively shown that the precise value of the Reynolds number at which flow ceases to be laminar, and the value at which turbulent flow becomes established, depends on the shape of the cross section. This was made especially clear in the work reported by Peter Feng.[28]

The fact that model flow intended to simulate turbulent prototype flow may tend to be transitional or laminar by virtue of the decrease in Reynolds number (inherent in the scale change) has long been recognized as a cause of difficulty in the verification of open-channel models.[29]

There is another possible explanation for Mr. Owen's finding f to be practically constant in the turbulent range. Other investigations have found f to decrease with increasing Reynolds numbers when the channel is smooth.[30]

[22] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP1151*, National Bureau of Standards, Washington, D. C., Vol. 21, December, 1938, p. 719, Eq. 48.

[23] "A Direct Optical Method for Measuring Fluid Velocities in Laminar Flow," by Ellis B. Pickett, thesis presented to the State Univ. of Iowa, Iowa City, Iowa, in 1950, in partial fulfilment of the requirement for the degree of Master of Science.

[24] "Experimental Studies of Viscous Flow in a Triangular Open Channel," by Sigurd W. Anderson, thesis presented to the Univ. of Minnesota, Minneapolis, Minn., in 1939, in partial fulfilment of the requirement for the degree of Master of Science.

[25] "Experimental Studies of Velocity Distribution at a Section in a 90° Triangular Channel," by Harry D. Purdy, thesis presented to the Univ. of Minnesota, Minneapolis, Minn., in 1947, in partial fulfilment of the requirement for the degree of Master of Science.

[26] "Flow in a Triangular Channel with Varied Boundary Roughness," by Joseph M. Forns, thesis presented to the Univ. of Minnesota, Minneapolis, Minn., in 1948, in partial fulfilment of the requirement for the degree of Master of Science.

[27] "A Study of Viscous Flow in an Open Channel Having a Circular Arc Cross-Section," by Daniel K. Donovan, thesis presented to the Univ. of Minnesota, Minneapolis, Minn., in 1949, in partial fulfilment of the requirement for the degree of Master of Science.

[28] "Experimental Studies of Viscous Flow in a Triangular Open Channel with Varied Central Vertex Angle," by Peter Chi-Te Feng, thesis presented to the Univ. of Minnesota, Minneapolis, Minn., in 1952, in partial fulfilment of the requirement for the degree of Master of Science.

[29] "Hydraulic Laboratory Practice," edited by John R. Freeman, ASME, New York, N. Y., 1929.

[30] "Resistance to Flow in Smooth Channels," by Ralph W. Powell, *Transactions*, Am. Geophysical Union, December, 1949.

Because the author states that the bottom of the channel had depressions and elevations of the order of 0.01 in., the question arises as to whether the channel was not acting as a rough channel. In Fig. 1 it can be seen that f has an average value, in the turbulent range, of approximately 0.034, which is equivalent to a Chezy coefficient whose value is 87. It has been found[31] that

$$C = 42 \log_{10} \frac{R}{\epsilon} \qquad (18)$$

in which C is Chezy's coefficient. A rough channel would give a value of C equal to 87 if $\frac{R}{\epsilon}$ were equal to 118. When R is equal to 0.15 ft, this would require ϵ to be 0.00127 ft or 0.0153 in.

HENRY MAKSOUD.[32]—Useful experiments have been conducted by Mr. Owen with the purpose of studying laminar and turbulent flow in an open channel and the transition from one type of flow to the other. The conclusions reached are, however, open to question.

Although no complete and consistent study has been made, as of 1953, of the mechanics of laminar and turbulent flow in open channels, the quotation cited by the author in the Introduction as indicative of the present state of knowledge on the subject is misleading. In a later edition of the text,[33] the quoted paragraph has been omitted as obsolete.

In Sect. 4, Mr. Owen states that the upper and lower values of the critical Reynolds numbers appeared to be the same for his experiments and were approximately equal to 4,000. This statement does not follow the accepted definitions of upper and lower critical values of the Reynolds number.[34,35] The lower critical value of the Reynolds number defines a condition below which all turbulence entering the flow from any source will eventually be damped out by viscosity. The upper critical value of the Reynolds number defines a condition above which laminar flow can never exist, and, therefore, the flow is always turbulent.

It is the lower critical value which is of practical importance because a moving fluid is seldom so free from local disturbances that laminar flow will persist at higher values of the Reynolds number. Above this lower critical value the flow can be either unstable laminar, or turbulent. Unstable laminar flow is that which might change to turbulent depending on the magnitude and frequency of the disturbances that might occur. The flow with a Reynolds number of 4,000 in the author's experimental channel could have been classified as unstable laminar flow because it could have changed to turbulent flow if the flow had been sufficiently disturbed.

Since G. H. L. Hagen (in 1854) indicated the existence of two states of flow which are termed laminar and turbulent, the phenomenon of transition from

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[31] "Resistance to Flow in Rough Channels," by Ralph W. Powell, *Transactions*, Am. Geophysical Union, August, 1950.

[32] Research Assistant, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

[33] "Hydraulics and Its Applications," by A. H. Gibson, Constable and Co., Ltd., London, England, 1952, p. 295.

[34] "Elementary Mechanics of Fluids," by H. Rouse, John Wiley & Sons, Inc., New York, N. Y., 1947, p. 174.

[35] "Elementary Fluid Mechanics," by J. K. Vennard, John Wiley & Sons, Inc., New York, N. Y., 1947.

one state to the other has been the subject of many studies, most of which concern flow in pipes. The results of much of the work differed regarding the true values for the lower and upper critical Reynolds numbers. The lower critical value for pipe flow is well defined. As to the upper critical value, attention is directed to the work of R. Comolet who experimented with flow in conduits of various shapes and reached values of the Reynolds number for circular tubes of 75,000 (50,000 was the highest value previously reached) for which the flow remained laminar.[36]

The author considered the flow in the 1.5-ft-wide channel to approach that in an infinitely wide channel. Consequently, Mr. Owen selected the depth of flow rather than the hydraulic radius as the length term in the Reynolds number and for the computation of the friction coefficient. One of the consequences of this approximation is the constancy of the friction factor in the turbulent region as shown in Fig. 1. Had the hydraulic radius been used, the results would not have deviated markedly from the Blasius relationship for smooth pipes.

In the subsequent computations, the writer uses the scant amount of data given by the author to show that this deviation is largely the result of the use of the depth in place of the hydraulic radius. For illustration, one point has been taken from Fig. 1 which has approximately the following coordinates:

$$\mathsf{R} = \frac{4\,V\,y}{\nu} = 130{,}000 \qquad (19a)$$

and

$$f = \frac{8\,g\,y\,S}{V^2} = 0.031 \qquad (19b)$$

A value of 0.3 ft is assumed by the writer for the depth y which according to the author is the maximum depth observed. The value of 1.2×10^{-5} for the kinematic viscosity is also assumed.

From Eqs. 19, the mean velocity was found to be equal to 1.3 ft per sec, and the slope was found to equal 0.000678. The hydraulic radius was computed from the known width and assumed depth, and with this hydraulic radius as the length parameter, the Reynolds number was found to be approximately 93,000. The friction factor was also recomputed and was found to be equal to 0.022. These values differ significantly from the values given in Eqs. 19. The new point, thus defined, if plotted in Fig. 1, would fall close to Blasius' relationship. This should be expected in the case of smooth channels having commonly used cross sections.

Had the author presented his data, a more complete analysis of the variation of the resistance coefficient with the Reynolds number could have been made. This analysis would be expected to indicate that, in the turbulent region, the Blasius and Kármán-Prandtl resistance equations for turbulent flow in smooth pipes serve as limits for the variation in resistance for open channels.

In the laminar region, the depths of flow are small, and the effect of using the depth of flow instead of the hydraulic radius is probably less marked. None-

[36] "Recherches sur la Genèse de la Turbulence dans les Conduites en Charge," by R. Comolet, *Publications Scientifiques et Techniques No. 246*, Ministère de l'Air, Paris, France, 1950.

theless, in the laminar region, the ratio of depth to width (or other shape parameter) is, as shown by Mr. Powell,[37] the significant criterion for determining the resistance law. In contrast, there is considerable evidence from experimental studies in smooth, open channels[27] that the resistance law in the turbulent region is essentially independent of channel cross section.

W. M. OWEN.[38]—It is not agreed that (as Messrs. Powell and Posey state) two errors are present in Eq. 3. The mathematical arguments which they have offered concerning the necessity for interchanging x and y can be shown to be incorrect. The assertion that the series in Eq. 3 diverges if $b > a$ is not substantiated by elementary convergence tests. As the absolute value of each term in the series is evidently less than $1/n^3$, the Weierstrass comparison test[39] shows that the series converges absolutely and uniformly throughout the rectangle and on its boundary, irrespective of the values of a and b. In fact, because the series $1 + 3^{-2} + 5^{-2} + \cdots$ can be used for comparison, the absolute value of the sum of the series in Eq. 3 is always less than $\pi^2/8$.

Obviously, the function defined by Eq. 3 satisfies the boundary condition, $w = 0$. Also, it can be seen by direct substitution that this function satisfies Eq. 1. Eq. 3 is therefore a solution of the boundary-value problem. Furthermore, it is the only solution as there is a theorem in potential theory which asserts that a problem of the Dirichlet type possesses only one regular solution.[40] How is this conclusion to be reconciled with the fact that the function proposed by Messrs. Powell and Posey (Eq. 15) is also a solution of the boundary-value problem? There is only one possible answer to this question. The two functions are identical; they merely have different representations. As the two expressions for w are equivalent, their integrals are equal. Consequently, Eqs. 5 and 16 are different representations of the same function, Q.

It is true that Eqs. 15 and 16 are better adapted for displaying the limits that are approached as b becomes infinite. An even simpler form of the equation for w is obtained if Eq. 15 is written as a sum of two series, and if the first series is evaluated by a Fourier series. Thus, there results

$$w = \frac{1}{2} K (a^2 - y^2) - \frac{16 K a^2}{\pi^3} \sum_{n=1,3,\ldots}^{\infty} \frac{(-1)^{(n-1)/2} \cosh \dfrac{n \pi x}{2 a} \cos \dfrac{n \pi y}{2 a}}{n^3 \cosh \dfrac{n \pi b}{2 a}} \quad (20)$$

If b becomes infinite, the series in Eq. 20 approaches zero, and only the terms that represent the solution for the infinitely wide channels remain.

As noted by several of the discussers, the writer erred in the assumption that the flow was two-dimensional when the depths in the channel were approximately 0.3 ft. It should be noted, however, that the depth only approached this value at the higher values of the Reynolds number and that the

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[37] "Resistance to Flow in Smooth Channels," by Ralph W. Powell, *Transactions*, Am. Geophysical Union, December, 1949, p. 878.

[38] Development Engr., Airesearch Mfg. Co., Los Angeles, Calif.

[39] "Theory and Application of Infinite Series," by K. Knopp, Stechert-Hafner, Inc., New York, N. Y., 1928.

[40] "Foundations of Potential Theory," by O. D. Kellogg, Dover Publications, New York, N. Y., 1953.

depths for laminar flow were extremely shallow; therefore, the 12.6% error, as given by Messrs. Powell and Posey, is considerably greater than that which actually existed. As cited by Mr. Iwagaki, Messrs. Powell and Posey, and Mr. Maksoud, the data on the turbulent-flow region would actually plot downward and to the left in Fig. 1—thus, close to the Blasius curve for smooth pipes.

Messrs. Powell and Posey state that a sentence in Sect. 3 is open to question:

> "* * * velocity distribution and energy loss for an open channel of depth a and width $2\,b$ (Fig. 3), are exactly the same as those of one half of a closed conduit of depth $2\,a$ and width $2\,b$."

They also note that the statement is certainly not true for turbulent flow. This statement was made in a section entitled, "General Equations for Laminar Flow in Channels of Rectangular Cross Section;" it was not implied that it pertained to turbulent flow. Messrs. Powell and Posey cite the fact that the point of maximum velocity in turbulent flow is depressed below the surface, thus making this assumption invalid for turbulent flow. Both the writer's data and those presented by Mr. Delleur indicate that the assumption is at least approximately true for laminar flow.

The statement in Sect. 2 that the channel showed depressions and elevations in the order of 0.01 in. should be clarified. Both the brass bottom and the glass sides of the channel were exceptionally smooth; there were points, however, where the channel surface deviated from a plane by approximately 0.01 in. This deviation would probably cause an error in slope rather than in channel roughness.

Mr. Delleur is correct in stating that, in addition to the fact that at the higher Reynolds numbers or at greater depths the flow could not be considered two-dimensional, there exists some doubt as to whether the flow was uniform. However, no velocity traverses were made. With respect to Mr. Delleur's conclusions concerning the distance required for the development of the boundary layer in the tests, it should again be noted that the 0.3-ft depth applies only to the highest Reynolds numbers.

Corrections for Transactions.—In Eqs. 1, 2, 3, and 4 the symbol ω should be changed to w. A similar change must be made in the boundary conditions for Eq. 2. In Eq. 3 the term $\left[1 - \dfrac{\cosh \dfrac{n\,\pi\,x}{2\,b}}{\dfrac{n\,\pi\,a}{2\,b}}\right]$ should be changed to $\left[1 - \dfrac{\cosh \dfrac{n\,\pi\,y}{2\,b}}{\cosh \dfrac{n\,\pi\,a}{2\,b}}\right]$. In Eq. 9 the symbol R should be changed to R, the hydraulic radius. The symbol R in Eq. 11 denotes the Reynolds number. Fig. 2 was inverted during the printing process.

DISCUSSION OF RIVER-BED SCOUR DURING FLOODS
PROCEEDINGS-SEPARATE NO. 254

E. E. DITBRENNER.—The paper by Messrs. Lane and Borland is a valuable contribution to the study of the regime of natural streams. As so frequently happens, a study of one of the stream characteristics in which we happen to be interested leads us into others. Until there has been some classification and acceptance of what we know, or think we know, about the laws governing the formation and maintenance of the channels made by streams flowing in them, however, it is doubtful if we can look for real progress in the final solution of any one of the elements.

After many years of working with and observation on streams in many parts of the world, some sort of a pattern seems to be developing in my own mind. The factual data presented by the authors seem to fit into that pattern.

In the author's explanation of the situation shown by Figure 3, it seems to me that the section depicting the crossing is oversimplified or else inadvertently misapplied. The section would not be rectangle—a bar would occupy the midsection. To explain this statement, I must digress a bit.

Geologists have, I believe, devised an accurate and useful classification of streams: youthful, mature, and old, based on its stage in the erosion cycle, the end of which produces a stream in perfect balance between work to be done and energy to do it—but with little work to do. This results in a stable channel in an alluvial valley the width of which is determined by the meander limits. These limits appear to be part of the regime of the stream—the larger the flow, the greater the meander limit, and the wider the valley. Geologists approach the matter from the standpoint of the transportation of sediment. The youthful stream, straight, at the bottom of a V shaped valley, is eroding downward, and needs more slope to carry the sediment. The slope of the stream remains related to its load of sediment. When it approaches or has reached its base level, it meanders and forms an alluvial plain as a valley floor. This can and usually does happen faster with the main stream than with the tributaries, so that frequently the former will lie in a trough or gorge, as does the Missouri and other streams in its course through the western high plains. The tributaries carrying less water, take longer to get to base level—to become mature and old. There are many variations and patterns, but the general explanation of the geologists appears to be sound, and no exceptions need to be made to the classifications they set up. Geologists do not explain what causes the meandering—and perhaps much more important—what limits it. Neither has anyone else done so. But Pettis' formula for the area of the flood prism in such a valley is very accurate. It would seem to follow some law.

In a very comprehensive series of laboratory experiments on meandering of streams, the Vicksburg Laboratory in the mid 1940's

determined a number of relationships between cause and effect, but in the end carefully refrained from any conclusions indicating an overall solution to the basic laws of stream regime. Without a doubt their one "firm" conclusion, however, was that when bank erosion ceases, meandering ceases. Lack of final conclusions was a great dissappointment to the originator and those who knew of the experiments, but certainly no fault can be found with the results of their work or the conclusions—so far as they went. In a visit with them some years back, however, they then confessed inability to explain some failures of revetment. Perhaps acceptance of the bank erosion conclusion should be conditional.

In a recent study leading toward a classification of factual data, it occurred to me that the one outstanding and universal characteristic of a natural stream is that it is a succession of pools and rapids—regardless of its size or age. This thought is by no means original, as I came across that idea in Colonel Charles R. Pettis' "A New Theory in River Flood Flow", published in 1927. I may have seen it before that. It checked my observation. Colonel Pettis confined his work to the larger streams, and my thinking stayed along those lines until recently.

In the stable channel, depicted by Figure 3 which has reached maturity in an alluvial valley of its own making, there is always a pool in each bend, the deepest part being just below midbend and toward the outside bank. The next bend is in the opposite direction. Between these two pools lies a bar, running diagonally across the channel from below and on the inside of the upper bend to the inside and above the lower bend. During rising stages, the pool is deepened and the bar is elevated. Any sampling of the sediment load between these points will certainly contain the load in transit between the pool and the bar on which this sediment is deposited, and easily lead to erroneous conclusions. The highest velocity is against the outside bank. It has the most erosive power. The velocity at the inside of the bend is low. Velocity distribution is far from uniform. At the "crossing", the velocity is more nearly uniform and the crest of the bar is apt to be nearly level, but highest in mid-stream. It usually does not tie into the banks on either end.

As the stage falls, the erosive power of the stream on the bar is less than in the pool against the outside bank, at least until the stage drops to the point where the bar becomes a weir, with a steep enough slope below to increase the velocity and scour effect. In other words—it forms "rapids". If the stream be navigable water, the top of this bar must be cut through by dredging before the stage drops too low, or navigation ceases. This is an old story on the Missouri, Mississippi, and other midwestern rivers. Sometimes the current can be trained with dikes to do this eroding.

A stream like the Rio Grande, as the authors state, does not impress one as having any of these characteristics. At what point the diagonal bar merges into a phenomena not so clearly recognizable is hard to say, but I am convinced that in fact the clearly defined diagonal bar becomes a transverse bar in "youthful" streams, and so stays until the sinuosity of the stream becomes great enough for clear delineation of the diagonal bar. You have the pool and rapid relation in some form. In the Phillippines on the larger streams, also "youthful" in character, the angle

between the bank and the bar was at times only 15°, but the bar was there! We had to find them, as they were indispensible fords.

In a recent study of scour around piers, it came to my attention that in many streams in New England, far from having to contend with scour at piers, there was a deposition of sediment around them! Old bridge piers were built on a timber grillage laid on the streambed. No excavating was done. Gradually deposition grew up around these piers and in many cases vegetation took hold and flourished. They withstood 50 or more years of ice and flood! Having grown up in the middle west, where scour always occurs around a pier, and is simply taken for granted, it was hard to believe, but there it was. Yet, in other locations on the same stream, this would not be the case.

From preliminary observations on a number of these crossings it appears that where the bridge crosses the stream in a "pool", there is no build-up around the piers. This "pool" may be in a reach (straight channel). Where the bridge is built across the stream over the "bar" there is build-up around the piers.

There are further differences of course, between New England streams and the Rio Grande and similar streams, especially in Maine. Mostly their headwaters contain large areas of lakes and swamps—essentially a reservoir. Precipitation is uniform throughout the year. Intensity of rainfall for any length of time under a week is lower than any other east of the 100th meridian. The result is that fluctuations in stage are generally not of great magnitude or rapidity, the banks and channels are stable, even though slopes are steep and channels are flat bottomed, wide and shallow—as is the Rio Grande and every stream with a steep slope, regardless of the sediment in which the channel is formed. The general lack of bank erosion may be significant. There may be simple explanations for these characteristics, and they are usually put forth, but too many times the pursuit of any one facet of a stream characteristic leads back to the same question: "What are the laws governing the formation, size, and shape of a channel made by the stream which occupies it?"

I think the author's facts raise the same question. This question has occurred to other people, of course, but so far no one appears to have the answer.

The availability of the data on the Imperial Dam station on the Rio Grande is fortunate. It points up a parallel situation, apparently little noticed. Almost all highway and railway crossings of major streams are over a bend, and in the edge of the valley! Of many thousand highway crossings of stream valleys I have studied, I presently recall only one which was in mid-valley. These crossings are all obviously in the "pool", which deepens as the stream rises. Scour around piers will be local and unless they be close together the scour will not join between piers. Therefore, the identity of the bed will not be lost. Conversely, however, if the entire bed is being lowered, measurement of the local scour caused by the pier can be in considerable error, since it will include the "pool" bed lowering as well as local scour. If the bridge was built across the diagonal bar, it is conceivable that local scour action might still occur, but with stream velocities low enough to deposit the

sediment from the pool, the deposition could easily counteract or cancel out the local scour. Since a valley crossing so designed as to cross over the diagonal bar would invariably have to be skewed adversely to overbank flood flow, we seldom if ever see or measure the phenomena of a diagonal bar from a bridge. This same pool and rapid conformation persists even into very small streams of very steep slope, on the very small one appearing essentially as sharp drops between flat slopes. This is one reason why a formula needs considerable "judgment" when applied to natural streams.

If we return to the classical bend and diagonal bar relationship, the main point raised by the authors' paper appears again. On such streams, the bends and the bar move progressively downstream. On smaller streams, old fords can be traced as having so moved. There is also a movement of the bed by means of saltation—a bed movement similar to that of sand dunes—where sediment is carried up a gentle slope and drops off over the ridge where the "lee" or downstream slope is steep. On streams like the Mississippi—these dunes are quite large. Their movement has been measured. If their rate of movement is a measure of the rate at which the bed sediments move, it takes some thousands of years for any given particle to move from St. Louis to the Gulf of Mexico. Any measure of sediment load will include some of this, some of the material moving from the "pool" to the bar, some from the bank erosion on the outside of the bend to the inside of the next bend—all strictly local freight. It is not in any sense a measure of the sediment which will be moved into and deposited in a reservoir or a delta. So far as I know no one has connected the diagonal bar with the saltation, but both are elements of bottom movement.

This does not appear to explain the phenomena of bed movement to great depth in these western mountain streams or similar phenomena in "straightened" midwestern streams. It does not, if such movement be considered to occur throughout the length of the river all at the same time. Since the bed movement to such great depths, however, is associated only with flow of great magnitude, which goes down stream in a wave, this bed movement certainly must be of the same character. It probably will not travel as fast or as far as the flood wave of water. Each succeeding flood will move this "wave" further on, and drop it as its energy diminishes. It moves slowly. People at the Keokuk dam have told me such waves pass down the Mississippi.

One of these waves is probably now at Albuquerque. If they are patient enough, without a doubt it will move on in time and the riverbed will again be below the city's streets. Unfortunately, no one can say how long this will be (granted the premise is correct) since probably no one has measured the bed of the river often, accurately, and at enough places to furnish substantial data to really confirm <u>any</u> theory.

One more facet of natural channels seems to fit here. The stream determines the dimensions of its channel. Only if a major change in regime occurs will the shape and size of the channel change. As it gnaws its way into one bank, the other bank closes in behind it. With all its scour, it remains the same size. Obviously then, only the surface erosion and the bed erosion will be available for deposit in a delta or a

reservoir. And by measuring this deposit, a true measure of bed lowering is available. On the Rio Grande, obviously, this is too small to measure accurately over a few years.

The paper serves to focus sharply the need for a great deal more factual data, more and accurate survey data of a continuing nature on more than one type of stream, and the continuous careful compilation, classification, comparison and analysis of all available data, in order to arrive at a concensus of what makes a stream tick. Maybe then we will not create new problems when we "solve" one.

It is also encouraging to see actual measured data indicating that every reservoir we build is not going to fill up before the contractor is paid off for building it. The general public has had too much of the other kind of data, without the painstaking research and honest presentation of Messrs. Lane and Borland.

EMMETT M. LAURSEN, A. M. ASCE AND ARTHUR TOCH, J. M. ASCE.[1]—This stimulating exposition of sediment movement in alluvial streams recalls an approximate analysis which was made by the writers to indicate the order of magnitude of scour to be expected at a contraction. This analysis, originally conceived as a guide to experimental studies of scour at bridge crossings conducted by the Iowa Institute of Hydraulic Research, should have direct application to the problem discussed by the authors.

A reach of a typical alluvial stream is represented schematically in Fig. 1. A portion Q' of the flood discharge Q of such a stream flows over the flood plain. The vegetation on the flood plain will reduce the velocity of flow and inhibit sediment movement. Therefore, the overbank flow will carry a negligible amount of sediment compared to the transport in the main channel. At contracted sections the sediment transport capacity will be dependent on the total discharge. Such conditions of discharge and transport can be described by Manning's equation of flow and DuBoys' equation of sediment transport capacity. These relationships, involving an extension of those used by L. G. Straub[2], M. ASCE, for simple contractions, take the form

$$Q - Q' = \frac{1.49}{n_1} b_1 d_1^{5/3} S_1^{1/2}$$

$$Q_s = C_s b_1 \tau_1 (\tau_1 - \tau_c), \quad \tau_1 = \gamma d_1 S_1$$

for the uncontracted section, and

$$Q = \frac{1.49}{n_2} b_2 d_2^{5/3} S_2^{1/2}$$

$$Q_s = C_s b_2 \tau_2 (\tau_2 - \tau_c), \quad \tau_2 = \gamma d_2 S_2$$

1. Research Engineer and Research Associate, respectively, Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City.
2. Straub, L. G., "Approaches to the Study of Mechanics of Bed Movement", Proceedings of Hydraulics Conference, State University of Iowa Studies in Engineering, Bulletin 20, 1940.

for the contracted section.

If the critical shear τ_c, is neglected as small compared to τ, simultaneous solution of the equations for the depth ratio d_2/d_1 results in

$$\frac{d_2}{d_1} = \left(\frac{b_1}{b_2}\right)^{9/14} \left(\frac{Q}{Q-Q'}\right)^{12/14} \left(\frac{n_2}{n_1}\right)^{12/14}$$

The first term on the right-hand side is the contraction effect determined by Straub, and the second and third terms embody the effects of over-bank flow and of roughness, respectively. Other assumptions could be made which might describe some situations more aptly, but similar relationships would result. For example, if the sediment movement on the flood plain is not negligible, but can be taken as a fraction Q_s' of the total sediment load Q_s, a fourth factor will be formed: $[(Q_s-Q_s')/Q_s]^{3/14}$

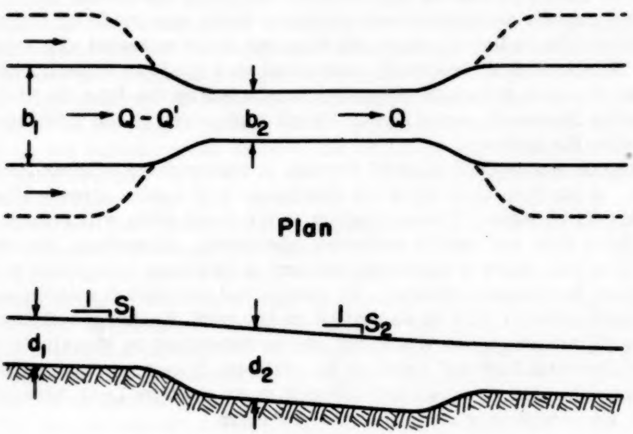

Fig. 1

This analysis refers to the equilibrium conditions which will obtain only if the flood is of sufficient duration. According to the analysis, neither the velocity of flow nor the sediment size affects the depth of scour at equilibrium. Laboratory experiments[3, 4] on scour around bridge piers and abutments confirm that the equilibrium scour is independent of the absolute rate of transport. The length of time required to attain, or closely approach, equilibrium will be dependent on the absolute rate of transport and, therefore, on the velocity of flow and the

3. Laursen, E. M., "Observations on the Nature of Scour", Proceedings of the Fifth Hydraulics Conference, State University of Iowa Studies in Engineering, Bulletin 34, 1953.
4. Laursen, E. M. and Toch, A., "A Generalized Model Study of Scour Around Bridge Piers and Abutments", Proceedings Minnesota International Hydraulics Convention, University of Minnesota, 1953.

sediment size. The rate of scour, of course, will be the difference between the rate at which sediment is supplied to the contracted section and the simultaneous excess capacity for transport which there prevails.

If, as the authors conclude, the material scoured from the contracted section does not go into suspension, deposition must occur in the next wider reach downstream (Fig. 2). Assuming $d_d L_1 b_1 = d_s L_2 b_2$, $n_1 = n_2$, and selecting arbitrary values of the ratios b_1/b_2 and $Q/(Q-Q')$ and of the low- and high-water depths d_1, Table 1 was prepared to show typical depths of scour that may be expected.

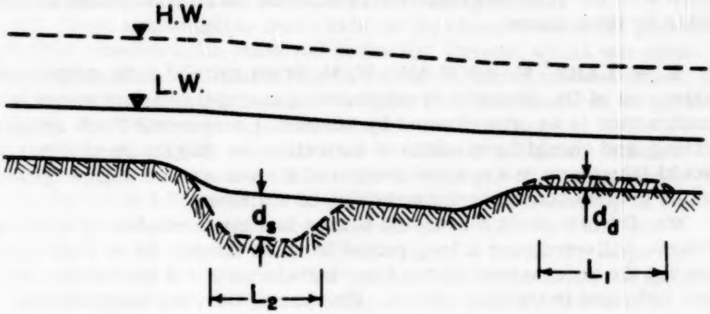

Fig. 2

Table 1

		Contraction Effect LW	Contraction Effect HW	Over bank Effect LW	Over bank Effect HW	Combined Effect LW	Combined Effect HW
b_1/b_2		2	2	1	1	2	2
$Q/(Q-Q')$		1	1	1	2	1	2
d_1		5	30	5	30	5	30
d_2		8	47	5	54	8	85
d_s	$L_1 = L_2$		9		12		35
d_d	$L_1 = L_2$		5		12		17
d_s	$L_1 = 10\ L_2$		13		22		49
d_d	$L_1 = 10\ L_2$		1		2		3

During the active scour and deposition period, scour should begin at the upstream end of the contraction, where the capacity for sediment transport is greater than the supply, and deposition at the upstream end of the following wide reach, where the capacity is less than the supply. Such an intermediate stage is shown foreshortened in Fig. 2. Both effects should progress downstream, approaching equilibrium, as the locations of the sections of excess and deficient capacity shift. A flash flood would not be of sufficiently long duration to move the scoured

material very far down the wide reach. If the length of the deposit were equal to the length of the scoured area ($L_1 = L_2$ in Table 1), sizable deposition would occur. A flood of longer duration would distribute the scoured material farther down the wide reach ($L_1 = 10\ L_2$ in Table 1), resulting in shallower deposit and deeper scour.

As shown in Table 1, the depth of scour which may be expected locally can readily be of the same order of magnitude as the rise in stage. Indeed, the combined contraction and over-bank effects for the example chosen result in a depth of scour almost double the surface rise. Thus the analysis gives an approximate quantitative solution which is in accord with the observed behavior of streams on alluvial plains as set forth by the authors.

E. W. LANE* M. ASCE AND W. M. BORLAND,** A.M. ASCE.—The extension of Dr. Straub's development of the hydraulics of scour in a contraction in an open channel by Messrs. Laursen and Foch is interesting, and should be valuable in indicating the magnitude of scour which would take place in a narrow section of a river or at a bridge opening, which greatly contracts the waterway of a stream.

Mr. Dittbrenner's discussion brings in a large number of facts on rivers, collected over a long period of observation. He is right in pointing out the unorganized state of our knowledge of the interaction of water and sediment in forming rivers. Because of the very large number of variables involved and the constant variation of many of them, this is inevitable. However, considerable progress is being made in this science, but for a long time to come there will be wide gaps in our knowledge. Mr. Dittbrenner's statement that a stream determines the dimensions of the channel is very true and recent publications[7] indicate many engineers are coming to this conclusion.

The Pettis concept of a river as a series of pools and rapids which Mr. Dittbrenner mentions is a useful one, especially for mountain streams, but care must be taken not to use it where it does not apply.

Figure 3 is admittedly a simplification, but it is believed that what it has left out does not detract appreciably from the soundness of the conclusions based on it.

Mr. Dittbrenner's contention that sediment travels in waves has some merit. Especially is this true in the wide portions of sand-carrying streams. Field studies of the Niobrara, Loup, and Middle Rio Grande Rivers all indicate a larger percentage of their total load moves by contact and saltation in the wide sections rather than in the narrow sections. The slower movement of these sand waves and dunes in the wide section of the stream accounts for considerable temporary storage of sediment

7. The Hydraulic Geometry of Stream Channels and Some Physiographic Implacations by Luna B. Leopold and Tom Maddock, Jr., Geological Survey Professional Paper 252.

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in these wide reaches of the river. However, the authors cannot conceive of a sand wave that would move as slow as the rate indicated by Mr. Dittbrenner in discussing the Albuquerque situation.

Mr. Stafford C. Happ, in a communication to the authors, has called their attention to an error in their presentation, in that they have not considered the overbank deposits in estimating the average depth of scour along the Rio Grande during a flood. He cites studies indicating that about two-thirds as much total volume (but twice as much sand) was deposited overbank as in the reservoir. However, he concludes that this does not weaken the authors' principal contention, namely that the whole bed of the river is not lowered during a flood.

Mr. Happ also supplies more data on the observation of 68 percent sediment concentration observed in the Rio Puerco, which was mentioned by the authors. Since this is, so far as known to the authors, a record concentration, it is important that its circumstances be put on record. The following is quoted from his letter:

> "The record 68% concentration from the Rio Puerco is based on a sample collected by the writer (Mr. Happ) at an estimated discharge of 2,200 to 3,200 cfs, 22 minutes after beginning of a flash flow which attained crest discharge of 3,000 to 4,000 cfs some 15 to 20 minutes later. This was not a relatively large discharge for the Rio Puerco. The sample was taken from the upper foot of water in a pint bottle having top opening of about 1-1/2 inches diameter. Mechanical analysis, after drying and subsequent dispersion, showed 75 percent of the sediment to be sand (larger than 0.625 mm). Preceding and succeeding samples had 57% and 64% sediment concentrations, of which 66% and 71% were sand, respectively. A dozen other samples during the receding stages, over a period of about 12 hours, showed consistent, progressive decreases in sediment concentration and percentage of sand.
>
> "Progressive changes culminating in concentrations of the order of 30-60% were measured during several other Rio Puerco flash flows, also with high but lesser proportions of sand. The proportion of sand generally increased with increase in sediment concentration. The flows of high sediment concentration were notably smooth, oily and streaming in appearance, and much less than usual surface waves, vertices and general appearances of turbulence."

The percentage is computed as the weight of the sediment divided by the weight of the sediment plus the weight of the water.

Since this paper was prepared, considerable information regarding the bed of the Rio Grande has been obtained by detailed surveys of a section of the river, made by the U.S. Bureau of Reclamation under the general direction of the junior author and immediate supervision of Mr. Ernest Pemberton. Unfortunately, no very high flows occurred during the period of observation. The results indicate that at the discharges observed, channels deeper than average extend from the narrow deep sections into the wider sections downstream, but these are not continuous and shift their position from time to time. If one tries to trace the

narrow deep channel downstream, he finds they become shallow and disappear altogether and if the river is examined at this point he can find the beginning of another narrow deep section within the wide reach of the river. The conclusion is that there are narrow deep sections within the wide shallow ones, but they are discontinuous. No continuous deep channel, such as suggested by Holmquist, was observed. Mr. Happ mentions the existence of a narrow, deep channel at Albuquerque during the 1941 and 1942 floods, and expressed the belief that this narrow deep channel was continuous, as suggested by Holmquist, but no conclusive evidence of this was supplied.

An analysis by Mr. Ernest Pemberton was made of data collected by the Corps of Engineers Albuquerque District at the Bernalillo gaging station in the Rio Grande during the 1948 flood at a section (width = 272 feet) that can be described as somewhat narrower than the average. The section did not start to deepen until a discharge of about 8,000 cfs was reached. At the peak discharge of about 13,000 cfs, the average scour was about 1.5 feet while the maximum scour was abcut 5 feet. It was concluded that narrow deep channels existed at periodic intervals throughout the length of the river. In the narrow reaches, these deep scour channels occur at the same location throughout a given flood but in the wide reaches the deep channels move from one location to another. Continuous sounding at the cable way showed a cycle of scouring and filling occurring with little variation of discharge which suggests bed load movement. It is believed that at such times the gradual and general changes in depth were caused by passing of a large sand bar and the local sharp lowering and raising of the bed level was due to movement of sand dunes.

In conclusion, it may be said that the principal contention of the author's paper, that there is no general deepening of the whole length of the rivers of the Rio Grande type during floods, seems to have been accepted by the discussors. Although the preponderance of the evidence is against the existence of a continuous narrow, deep channel in the Rio Grande, such as suggested by Mr. Holmquist, the extent of the evidence is insufficient to preclude the possibility that it does not happen under some conditions.

DISCUSSION OF HIGH VELOCITY TESTS IN A PENSTOCK PROCEEDINGS-SEPARATE NO. 297

STEPONAS KOLUPAILA*—Investigation in penstocks presented in this paper are of high value as they reached unusually great Reynolds' numbers, to $3.8 \cdot 10^7$. In the literature there are few examples with these values as large as $1 \cdot 10^7$. An extensive summary of Italian data was published in 1936 by the Commissione per il Controllo del Funzionamento di Grandi Opere Idrauliche (Ref. 1). The greatest Reynolds' number mentioned there is $1.08 \cdot 10^7$, for a riveted steel pipe 2.80 m (102") in diameter.

Velocity distribution in these unusual conditions is particularly interesting. As author shows in the Fig. VIII his results disagree with the known theoretical laws of velocity distribution. Both the logarithmic law by Prandtl and the combination of a parabola with a logarithmic curve by von Kármán give essentially larger values than the observed data. A simple parabolic law, widely applied by Nikuradse, $v/v_m = (y/r_o)^x$, with the power x about 0.1, would fit evidently better.

The writer examined the complete results of one representative test, copy of which he obtained by the courtesy of Mr. Burke. Original data were slightly corrected eliminating several pulsation humps. For further plotting the averages were computed for the velocities in the same distance from the pipe center. A summary of these valuable data is presented in Table A:

In order to prove the law of velocity distribution the averaged velocities v are plotted against the distance from wall y : 1) on the semi-logarithmic net to check the logarithmic law, 2) on the logarithmic net to check the parabolic law. Both plottings have the same vertical scale and adjusted horizontal scale for nonpartisan conclusions. The points below $y = 0.01$ ft deviate in both cases: a small difference in distance causes here a large relative error.

The logarithmic curve shows a clear break at a distance of 0.33 ft or at 15.6% of the radius. The parabolic curve seems to be much more dependable; its equation is $v = 31.9\ y^{0.0857}$. The low power $x = 0.0857$ corresponds well to the smooth pipe surface and to the small value of the Manning coefficient $n = 0.0096$. Empirical relationship between the power of parabola and the Reynolds' number, based on earlier European data, $1/x = 2 \log \underline{R} - 2.12$, though far extrapolated, fits surprisingly well with this test.

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TABLE A—Velocity Observations in the 51" Penstock

Distance from wall ft	Velocities along four radiuses, ft/sec A	B	C	D	Average
0.002	18.12	20.09	18.80	17.07	18.52
0.004	20.17	21.89	20.66	19.89	20.65
0.008	20.80	22.48	21.28	21.37	21.48
0.013	21.82	22.58	21.85	22.27	22.13
0.017	22.22	22.97	22.49	22.74	22.60
0.021	22.60	23.17	23.35	23.12	23.06
0.025	23.15	23.62	23.17	23.30	23.31
0.029	23.35	23.91	23.82	23.86	23.74
0.033	23.62	24.36	24.01	24.04	24.01
0.038	23.99	24.41	24.28	24.21	24.22
0.042	24.25	24.55	24.38	24.21	24.35
0.046	24.45	24.64	24.74	24.74	24.64
0.050	24.60	25.00	24.94	24.83	24.84
0.054	24.77	25.00	25.19	25.00	24.99
0.058	25.03	25.01	25.27	25.26	25.14
0.063	25.20	25.19	25.45	25.26	25.28
0.067	25.25	25.27	25.55	25.51	25.40
0.071	25.28	25.31	25.62	25.81	25.51
0.075	25.70	25.38	25.72	25.92	25.68
0.079	25.61	25.45	25.83	26.09	25.74
0.083	25.86	25.46	26.06	26.20	25.89
0.100	26.26	25.96	26.32	26.40	26.24
0.117	26.41	26.32	26.50	26.88	26.53
0.133	26.89	26.67	27.01	27.10	26.92
0.150	27.20	26.81	27.31	27.34	27.16
0.167	27.42	26.94	27.44	27.64	27.36
0.208	27.54	27.27	28.03	27.85	27.67
0.250	28.05	27.82	28.61	28.05	28.15
0.300	28.55	28.40	29.03	28.78	28.69
0.400	29.14	29.14	29.82	29.47	29.39
0.600	30.36	30.09	30.88	30.57	30.48
0.800	31.12	31.07	31.74	31.62	31.39
1.000	32.00	31.82	32.56	32.25	32.16
1.200	32.58	32.49	33.12	33.13	32.83
1.400	33.11	33.01	33.40	33.47	33.25
1.600	33.80	33.46	34.01	34.00	33.82
1.800	34.04	33.97	34.21	34.19	34.10
2.000	34.14	34.21	34.23	34.20	34.20
2.114	34.33	34.33	34.33	34.33	34.33

Data: $v_m = 30.0$, $d = 4.229$; for water at 55.5°F kinematic viscosity $\nu = 1.298 \cdot 10^{-5}$, $\underline{R} = 9.78 \cdot 10^6$, $\log \underline{R} = 6.990$,

$$1/x = 2 \cdot 6.990 - 2.12 = 11.86; \quad x = 0.0843 \ .$$

This power can be further checked by the ratio between the average velocity across the pipe and the velocity at the center:

$$k = v_m/v_o = 30.0/34.3 = 0.874;$$

this value is unusually high.

For the parabolic law $v = a\ y^x$ integration across the pipe gives ratio $k = 2/(x+1)(x+2)$. Substituting $x = 0.0857$, $k = 0.883$, or, substituting $k = 0.874$, $x = 0.0932$. Both results are close.

The discharge for these calculations was determined graphically by plotting $v\ (r - y)$ against y and planimetering the area beneath the curve. The area was 67.10 and the discharge $Q = 2\pi \cdot 67.10 = 422$ cfs, about 3.1% less, than recorded 435 cfs; this difference seems to be too great.

$$v_m = Q/A = 422/14.08 = 30.0 \text{ ft/sec} \ .$$

On the same occasion the Coriolis' coefficient $\alpha = \int v^3 dA/v_m^3\ A$ was determined by plotting $v^3(r - y)$ against y. The planimetered area was 62,200; $\alpha = 2\pi \cdot 62{,}200/(30.0)^3 \cdot 14.08 = 1.035$.

This value corresponds well with the usual limits for smooth pipes (Ref. 4).

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FRANK B. CAMPBELL M. ASCE.[1]—The tests results presented by the author are particularly valuable in view of the lack of experimental work on flow in pipes or conduits with large Reynolds numbers. The Reynolds numbers in the 123 inch line extended to 3.8×10^7. A number of experiments have been made with R values somewhat less than 10^7. The R values of the Ontario Power Company test[2] reached 3×10^7.

1. Chief, Hydraulics Analysis Branch, Waterways Experiment Station, Corps of Engineers, Vicksburg, Mississippi.
2. "The Flow of Water in Concrete Pipes," by F. C. Scobey, U.S. Department of Agriculture Bulletin No. 852, October 1920.

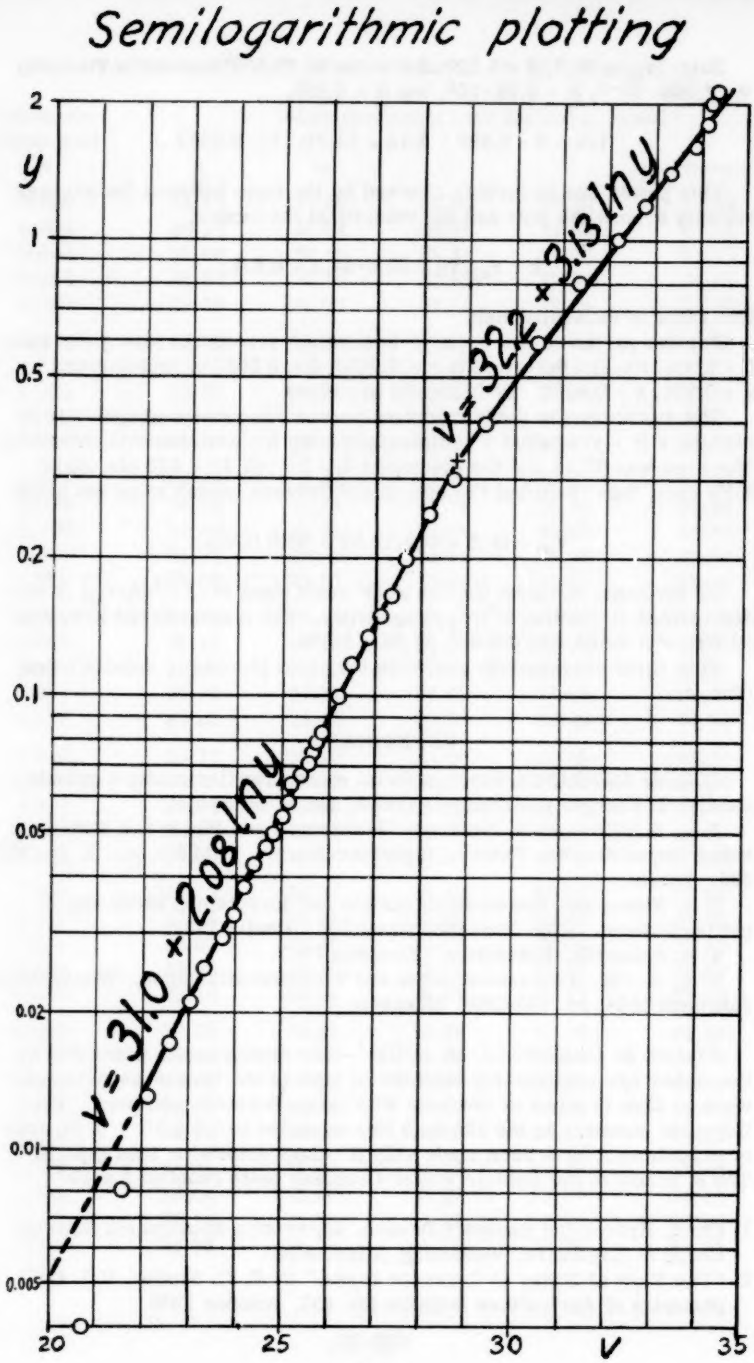

Semilogarithmic plotting
y
2
1
0.5
0.2
0.1
0.05
0.02
0.01
0.005
20
25
30
35
V
V = 32.2 + 3.13 ln y
V = 31.0 + 2.08 ln y

Fig. 1

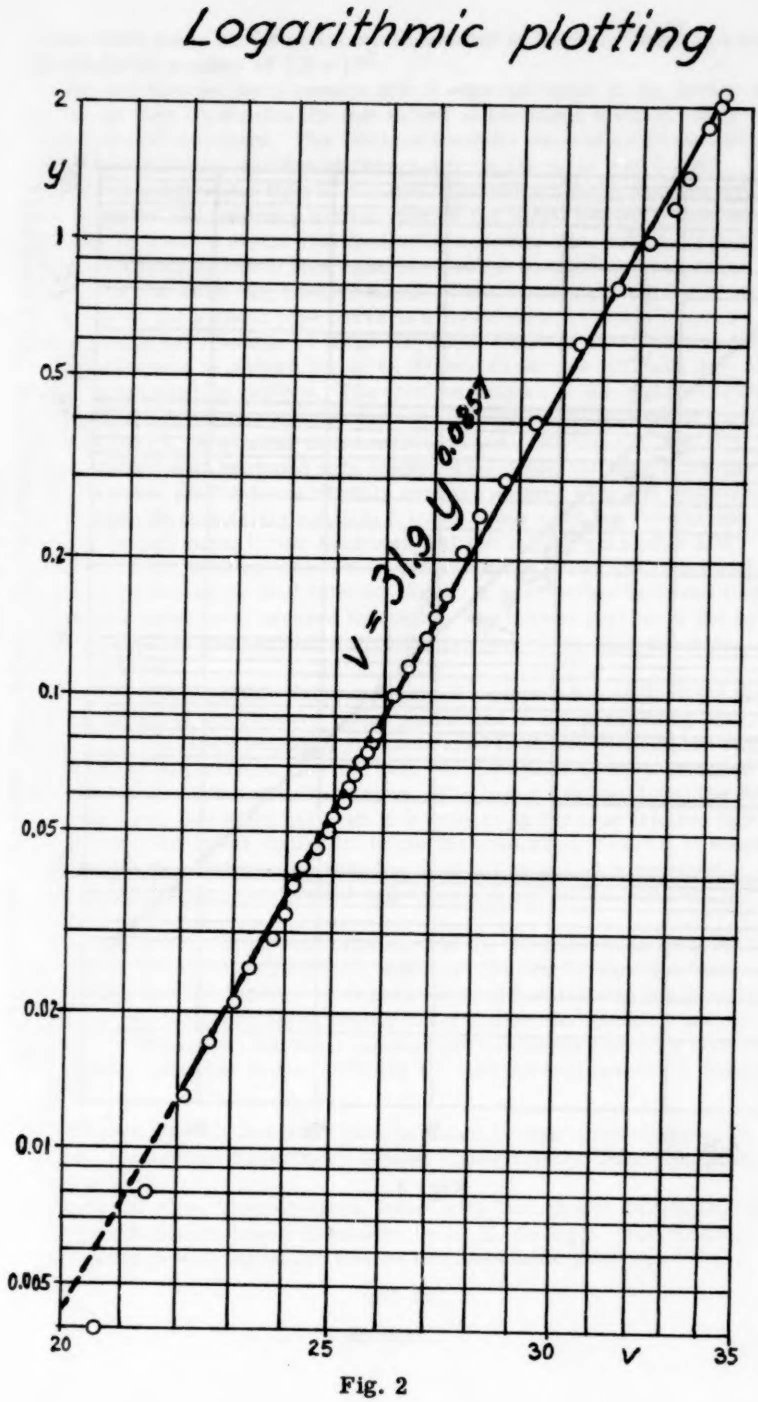
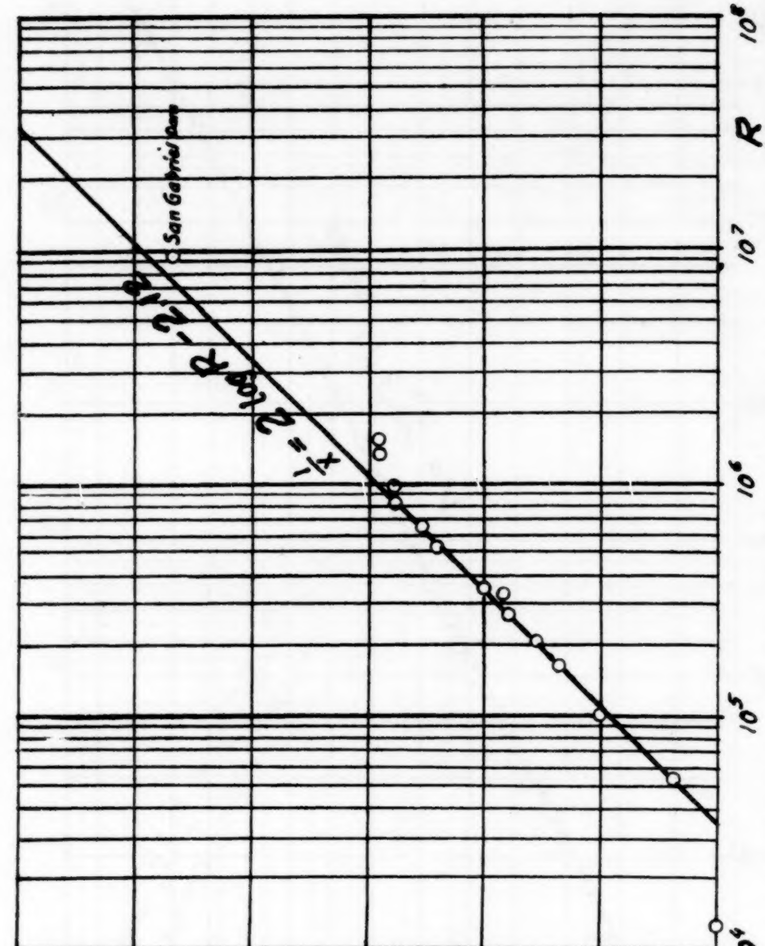

Logarithmic plotting
y
2
1
0.5
0.2
0.1
0.05
0.02
0.01
0.005
20
25
30
35
V
V = 31.9 y^0.0857

Fig. 2



Relationship between the power of parabola and the Reynolds' number in smooth pipes

Fig. 3

Tests were made on the outlet works conduit at Denison Dam[3] at a single Reynolds number of 1.2×10^8.

The San Gabriel tests results are of especial value to the design engineer as they emphasize the low values of Mannings n which occur at high Reynolds numbers. The Denison conduits were of concrete poured against steel forms and the Mannings n from the tests was 0.0097. This is a much lower value than is commonly used for design assumptions and indicates the tendency to over-design for large Reynolds numbers. The Darcy friction factor for the Denison conduit was 0.0064. The Von Karman-Prandtl smooth pipe equation gives a Darcy f value of about 0.0058 for the same Reynolds number. Thus it appears that the trend of values of the smooth pipe curve is a more reliable guide to design assumptions for conduits of large Reynolds numbers than is Mannings n.

The grouping of plotted points on Figure III for the 123 inch pipe reveals an interesting pattern. The friction factors at the higher Reynolds numbers are generally greater for the 700 foot reach than those for the 500 foot reach. A similar phenomenon was observed in the Denison conduit which was equipped with piezometers along the entire length. The pressure gradient was slightly concave upward near the upstream end and exhibited a fairly straight friction slope near the downstream portal. In this case, it was reasoned that the intake, gate slots and transition generated an intensity of turbulence in excess of that which would be sustained by wall friction alone. It was further believed that the excess turbulence decayed throughout the curved portion of the hydraulic gradient and reached a uniform friction slope near the downstream portal.

The pattern of plotted data on Figure III appears to substantiate the interpretation of the Denison tests, mentioned in the preceeding paragraph. As the reach lengths were measured from the piezometer ring furthermost downstream, the 700 foot reach could well have extended into an excess turbulence decay zone. The lower f values from the 500-foot reach are therefore believed to be closer to the true friction factor. Even though the lower values lie below the smooth pipe curve, it seems remarkable that an equation based upon experiments at substantially lower Reynolds numbers should extrapolate to values so close to the author's results.

The generation of excess turbulence at the pipe entrance believed to occur only at very high Reynolds numbers. Dr. G. H. Keulegan[4] has pointed out that for laboratory experiments with relatively low Reynolds numbers, the boundary layer begins to form near the entrance and increases in thickness, leaving a laminar core extending into the pipe. It can be seen from the author's Figure III, that the test points for Reynolds

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3. Pressure and Air Demand Tests in Flood Control Conduit, Denison Dam, Waterways Experiment Station Miscellaneous Paper No. 2-31, April 1953.
4. Theory of Flow Through Short Tubes with Smooth and Corrugated Surfaces and Square Edged Entrances by G. H. Keulegan, The Highway Research Board, Research Report 6B, December 1948, pp. 9-16.

numbers smaller than 2 x 10^7, exhibit no such pattern as can be found for the test points at higher Reynolds numbers. It is conceivable that for the entrance geometry under consideration, a laminar core extends from the entrance for Reynolds number less than this value, but that excess turbulence is generated near the entrance for larger Reynolds numbers.

Verification of this supposition may be accomplished only when there is opportunity to measure and compare the velocity distribution near the entrance with that for a section much further downstream. The development of apparatus to allow the turbulence spectra for these two locations to be compared may also offer information on the generation of excess turbulence for high Reynolds numbers.

The question of excess turbulence and its decay is important not only in the proper measurement of the friction factor but also in the evaluation of the entrance loss coefficient. More information is needed on the intensity of turbulence near the entrance so that means can be devised to simulate the condition in models of large conduits with high velocities.

ARTHUR L. COLLINS[1], A.M., ASCE.—The measurements made on the San Gabriel Dam penstocks of 51", 96" and 123" by Maxwell F. Burke[2] have produced data which is of particular interest on account of the velocities which were made available. Venturi tubes of an unusually large throat ratio, .82 and .93, were calibrated. A careful consideration of the data relating to pipe friction and venturi coefficients shows the need of a Code not now in existence for conducting tests.

DISCUSSION

The purpose of this discussion is to offer suggestions for improving the methods of making field measurements, and not to enter into a discourse on the mathematical formulas of Prandtl, Karman and Reynolds. The first objective of an exploration is to obtain accurate data. With that information the student can refer the data to the hydraulic charts such as are authored by Fred C. Scobey[3] or the Hazen and Williams tables, to see where the information fits into the general picture, and can be put into practical use.

Pitot Tubes

The conditions at the San Gabriel Dam were very favorable for the use of the pitot tube as a means for determining accurate discharges. The use of one pitot static tube held in one position as an independent unit to serve as a pilot, and a multiple tube system for obtaining the velocity distribution, offers certain advantages. The logical tube to use in view of the high velocity is designated as the STANDARD PITOT-STATIC, for which the National Advisory Committee for Aeronautics[4] must be given credit. The design of the tube is based upon an extensive study of pitot tubes conducted in 1935. While the tube has been used in the Code[5] for testing fans, it has been largely ignored in hydraulic circles.

ASME

The ASME has been the principal source of information relating to water measurements which include use of the pitot tube. The writer considers the ASME Power Test Code, 1949,[6] impractical in the matter of pitot tubes; and he believes that the manufacturers of turbines and pumps have been partly responsible for this condition. The above statement is not intended as a slur at the integrity of the manufacturers, but is only meant to call attention to the hands-off policy regarding acceptance tests. The attitude would be entirely different if there were a method of measurement which could be depended upon to measure with an error never exceeding 1%.

In the above Code the two accepted methods are that of a particular commercial brand, and the old method using a single orifice tapered tip about 3 in. long attached to a rod. The static piezometer is built into the former rod, while in the latter the static must be obtained at the pipe wall.[7, 8] If the making of a test was so complicated by having to make coefficient corrections up to 3%, as outlined in the Code, and there was no simpler method, then there should be no recognized pitot tube method of measurement. The required tolerance of 1% is obscured when the arbitrary corrections are applied.

Hydraulic Institute[9]

This is a technical agency supported mostly by pump manufacturers in contrast to the turbine or water wheel builders. It has its own Code which, for example, is used by the U.S. Bureau of Reclamation. However, the Code is principally that of the ASME. It specifies the use of the Gibson, Allen salt-velocity method, and the Cole method. The remaining tapered tip single pitot method is self-eliminating because of the controversial factors entering into the data.

The Code recommends that the test be conducted by the inventors of the methods, or experts with the methods, which amounts to the same thing. The manufacturer of the turbine benefits by a short water measurement, while the pump is favored by a long count.

It has been the practice for several years to escape the hazards of making an erroneous field measurement by the buyer accepting the laboratory test of the prototype, in lieu of the field test.

During the past year or so the U.S. Bureau of Reclamation has endeavored to improve the Allen method. Thus it would appear that the method is not infallible after some twenty-five or thirty years in the hands of experts.

Technical Notes 546[10]

These notes cover the analysis of 9 pitot tubes which were in general use by various important agencies. With this information one is able to anticipate the performance of any type of pitot tube of the conventional bent tube type. It does not refer to the pitot cylinder or capillary tube. The information has made it possible to construct what is called the

Standard as referred to previously. It consists of an outer tube 5/16" in diameter with a small inner tube. The tip or nose is semi-spherical, impact orifice 1/16", static openings are drilled with a #60 drill, and spaced 2 1/2" from the tip or nose. The length is 14".[11] The tube can be attached to a handle or stem, or to a support.

The coefficient is unity, and is compensated in that angularity has little effect on the readings. Similarly made tubes will agree within 1/4 of 1%. Its manufacture is not controlled by patent. The design is suggested by the U.S. Bureau of Standards, who will check its calibration for a nominal sum.

Two methods for the use of the tube are suggested. First is the spacing of at least 10 tubes at specified radial distances on a frame across two diameters. It is the conventional method of dividing the pipe area into annular spaces of equal areas. Next is the use of 4 pitots on a handle or stem, and which are introduced at 4 places on the pipe circumference.

The Aeolian vibrating effect on the tube at the high velocities encountered at San Gabriel is always a problem, and must be prevented. This term has been borrowed from the text of a U.S. Patent which describes a method of measurement using a rod that is suspended from a wall, and which emits vibrations.

Plotting of Data

In any type of tube where a traverse is made the user is confronted with the difficulty of interpreting the manometer readings taken near the pipe wall. The method of dividing the pipe area into equal annular areas, and omitting the reading of the velocity in the area close to the pipe wall, has given very satisfactory results. The plotting of the data with the velocity on the vertical scale, and the radial distance "squared" on the horizontal, offers an aid in arriving at the proper rating when the curve is projected to the wall. The outer annular area velocity is not eliminated in this method, but the effort to make a reading within a fraction of an inch of the wall is unnecessary. This outer area is always unstable from turbulence of such a nature that is practically impossible to tie the readings in with the tube calibrations.

Pilot Pitot

In the planning of tests as conducted at San Gabriel it would be of considerable interest to install a pilot tube either of the Standard or the pilot cylinder type some considerable distance ahead of the venturi, with the orifices near the mid-diameter. From the general data now accumulating regarding the velocity distribution in large pipes, it is apparent the average pipe velocity ratio is about 85 1/2, with variations of from 82 to 89. In the information on ratios the type of pitot is never mentioned. Hence it is impossible to tell if the results given are comparable to the extent of 3 or 4%, due to the characteristics of the tube.

The Cole Pitometer[12] has been in use for over 50 years. The pitot manufactured by the Simplex Meter Company[13] has been in service for

over 30 years, and the pitot cylinder for a similar period. While these tubes serve a useful purpose, it will be found that a velocity traverse made in the same pipe section by these types of tubes will not show identical curves. Also it can be observed that a tube used by inserting the same into the pipe and extending it entirely across the pipe will show one curve. If the test is repeated by inserting the tube from each side of the pipe and extending it to the mid-diameter, the two curves will not be identical. The Standard tube has its orifices far enough away from the supporting stem to avoid interference with the readings. This tube with its other characteristics makes it the choice of all the tubes now available.

Consistency

When the pilot tube is put in service the pipe velocity coefficient is assumed to be 85 1/2%, with the possibility that the correct value is 3 1/2% in error. It can be further assumed that the same error will hold over the entire range of velocities from about 2 fps and over, and the consistency will be not greater than 1/2 of 1%. However, these are assumptions. If during the tests for the venturi coefficient or for any other purpose it is possible to obtain a single accurate check on the assumed values, the pitot becomes an accurate metering unit. This is also predicated upon the assumption that the velocity distribution curve follows the parabola, except for velocities of one or two fps.

A large water distributing company has used this method for several years, having installed over forty pitot tube installations with recorders on cement lined pipes from 8" to 48" in diameter. It was found unnecessary to calibrate the tubes in the field. The tubes forming the metering system are primarily for control purposes, and no serious harm will result if the rating is 3 or 4% in error.

Venturi

In the 51" venturi data it is noticed that the average coefficient is .961, with a spread of 4 1/2%, and the readings do not show a trend to change from high to low velocity. The inference is that the velocity or distribution ratio of the flow of the water in the pipe does not change. Had there been installed a pilot tube and readings taken simultaneously with the venturi, the data could have been analyzed to find the cause of the variations.

It would appear that the reading of greatly depressed manometer deflections up to .001" is superfluous, in view of the more important details incidental to the overall accuracy.

Capillary Pitot

In the capillary tube data obtained from the 51" pipe, it will be interesting and perhaps instructive to plot the original manometer readings without any eliminations. The length of the capillary pitot tube is but 1" long, and the tube is supported by a relatively close and large cross

member. The support members themselves may have some effect on the readings, and particularly the ones taken from the trailing member. The curve should be plotted from the square root value of the deflections and the square of the radial distance. How closely will the curve approach a straight line? There will be a 1 or 2% area at the wall which must be properly evaluated. In routine testing using the equal ring or annular area method, the outer radius for the last reading is but 5.1% of the radius from the wall. In the 51" pipe this amounts to 1.3 inches. It is generally assumed that the continuation of readings at small intervals beyond this distance is not justified. The projection of the curve to the wall is always of interest.

Dye Method

The application of the color test for conditions existing at San Gabriel appears to check out very well. The method should provide consistent readings for the same reasons that hold true for the venturi and pilot tube. Consistency again does not mean accuracy. In a color test if consistency does exist, it is only necessary to apply a correction as is done with the pitot tube in order to obtain accurate data.

Pitot for Small Pipes

While the discussion of the Standard tube has been stressed, its necessarily long length makes it awkward to use in a small pipe. There are 100,000 or more irrigation pumps in California with discharge pipes of 6" to 12" in diameter. These pumps should have checks at intervals to determine the efficiency and amount of water pumped. The tolerance accuracy should be within 2 or 3%. This can be accomplished by the cylinder pitot. This type of tube is described in Agricultural Engineering[15] from data originating at the University of California Agricultural College at Davis, California.

The P. G. & E. Power Company in 1933 became greatly concerned with the efficiency of the farmers' pumps because of the economic effect of insufficient water and excessive cost of power, where the pump for some reason was wasting energy. The company then established a free testing service through a special department which has continued for over twenty years. During this time over 50,000 tests have been made.

Notwithstanding the wide use of the cylinder pitot, the engineering student is led to believe the weir and the related Parshall flume and orifice plate are the established methods of measuring water in the field. The orifice plate is of course satisfactory where conditions permit. However, in 50,000 power company tests it is doubtful if the weir would ever be used where the pipe could be tapped for the pitot. Ordinarily an accurate reading can be made in a few minutes time under practically any condition met with in ordinary pumping plants.

The blight hanging over the full use of the pitot shows itself in the Water Measurement Manual[14] published and released in 1953 by the U.S. Reclamation Service. It contains 91 pages devoted to the weir, Parshall flume and submerged orifice out of a total of 134 pages. One page simply

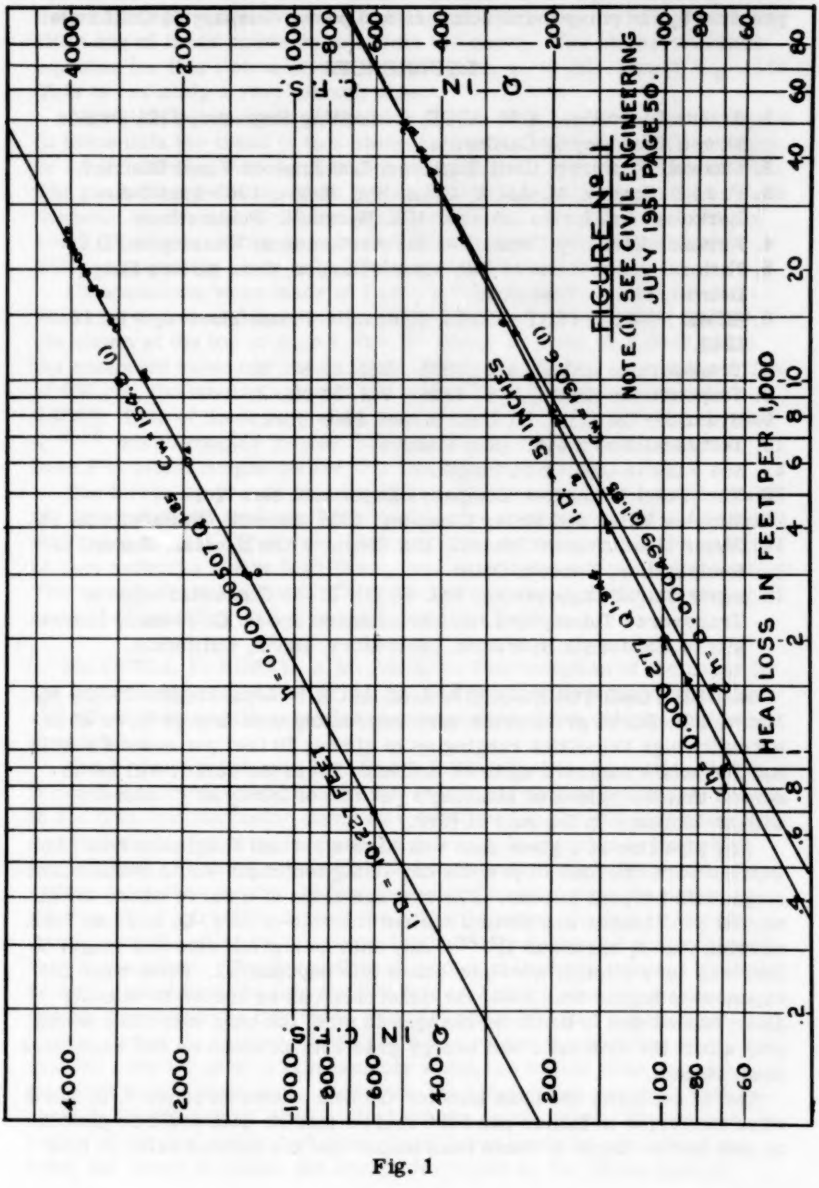
FIGURE No 1
NOTE (1) SEE CIVIL ENGINEERING JULY 1951 PAGE 50.
HEAD LOSS IN FEET PER 1,000
Q IN C.F.S.
I.D. = 10.22 FEET
h = 0.0000507 Q^1.85 C=154 (1)
I.D. = 51 INCHES
h = 0.00499 Q^1.85 C=130 (1)

Fig. 1

states the original formula discovered by Pitot, and a reference to the Fluid Meter section of the ASME, with no reference to the methods in practice by the pump manufacturers and power company in California.

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5. National Association of Fan Manufacturers, Gen. Motors Bldg., Detroit, Mich. Test Code.
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15. Agricultural Engineering, Vol. 18 (1); 21-24 Characteristics of Transverse Tubes, by J. E. Christiansen and O. C. French, University of California Hydraulic Laboratory, Davis, California.

R. CLIFFORD YOUNGQUIST, A.M. ASCE.[1]—Separate No. 297 by Mr. Maxwell F. Burke gives some very interesting test data on flows of abnormally high velocities ranging up to almost 50 feet per second which had Reynold's numbers up to 38 million. From the data it will be observed that the values of Manning's "n" and of Darcy's "f" decrease with an increase in the rate of flow.

Any pipe line at a given date will have a certain roughness condition which before the time of present day lining technique would become more rough with respect to time. The average slope of a curve where the observed head losses are plotted against the rate of flow, Q, is about 1.85, showing that h_f varies as $Q^{1.85}$. Any data on a given size and length of line may vary slightly above or below this exponential. Often when the exponent is higher than 1.85, the difference can be traced to velocity disturbances due to bends or changes in size, the head loss from which may affect the hydraulic and energy gradients as much as 100 diameters down stream.

Out of curiosity the head loss for the data shown on pages 7, 8, and 9 were converted to losses per 1000 feet so that all data could be plotted on one curve. Some of these head losses and the related rates of flow

1. Hydraulic Engineer, Dept. of Water and Power, Los Angeles, Calif.

were plotted on Figure No. 1 followed by a line having a slope of 1.85 drawn through points for the 123 inch pipe. It is interesting to note how little any of these points depart from the curve. The Hazen-Williams equation for this line is shown on the figure, as is the value of $C_w = 154.8$. This is certainly a very smooth pipe.

Likewise, points were plotted for the 51-inch line on the same figure. In these data the trend is to a steeper slope or exponent which amounts to 1.944, showing that for this pipe line the head loss varies as $Q^{1.944}$. The second curve is based on Hazen-Williams equation passing through the point where $h_f = 49.86$ feet and $Q = 505$ cfs. This line has a C_w of 130.6. It is interesting to note that for a flow of 200 cfs the Hazen-Williams equation gives only 0.9 foot more of head loss.

Computations were made of Darcy's "f" comparable to the Hazen-Williams equation shown for the 123-inch pipe. For the flow of 4100 cfs shown at the top of page 8, the "f" factor is given as 0.0063 while the computed value of "f" is 0.0065. Using the data on page 7 for a flow of 600 cfs, the tabular value of "f" is 0.0083 which was computed at 0.0086. Both of these sets of figures show that the "f" values vary as $Q^{-0.15}$, which roughly proves that the Hazen-Williams equation will have a constant roughness for this line amounting to $C_w = 154.8$.

The foregoing proves that the Hazen-Williams equation may be used for pipe networks analysis for velocities as much as ten times the normal rates of flow. High velocities are often encountered in the analysis of pipe networks where high flows, real or imaginary, are being studied. The results from the Hardy Cross or the new rapid electrical analog method of modeling will give very excellent values.

MAXWELL F. BURKE, A.M. ASCE.[1]—The reception of the paper by the engineering profession is highly gratifying to the writer. The time and effort necessarily expended in preparation of the discussions are appreciated.

The writer agrees with Mr. Frank B. Campbell's suggestion that the friction factors developed in the 500 foot test reach are probably closer to the true friction factor than those developed in the 700 foot reach, and that excessive turbulence at the entrance to the pipe is probably responsible. Factors responsible for such excess turbulence are transitions from the common 30 foot diameter tunnel to pipe diameter, expansion and contraction around the butterfly valve, the butterfly valve itself, vacuum relief valves below the butterfly valve and manhole openings in the pipes. The possibility of excess turbulence created by these factors extending down into the 700 foot test reach was the primary reason why pressure taps were installed for a 500 foot reach also.

Mr. Arthur L. Collins, A.M. ASCE has raised the question of a standardized code for pitot measurements which the writer does not feel qualified to comment upon. However, the information supplied with respect to the "Standard Pitot-Static" tube is appreciated. The writer was aware of several designs of pitot-static tubes made by competent persons, but chose to utilize the design developed by the Flood Control

1. Civ. Engr. Los Angeles County Flood Control Dist., Los Angeles, Calif.

Tabular Material

Q Pitot	a	x	V Max.	V Mean	V Mean/V Max.	k
96.24	7.29	.0948	7.88	6.85	.742	.872
114.8	8.71	.0976	9.40	8.17	.869	.869
124.9	9.49	.0965	10.26	8.89	.867	.870
142.3	11.33	.0937	12.15	10.55	.868	.873
161.4	12.25	.0956	13.24	11.49	.868	.871
165.9	12.70	.0960	13.61	11.81	.868	.871
198.8	15.20	.0998	16.35	14.15	.865	.866
238.3	18.11	.0980	19.42	16.96	.873	.868
252.9	19.20	.0924	20.52	18.01	.878	.875
278.5	21.31	.0910	22.63	19.83	.876	.877
298.3	22.64	.0889	24.17	21.24	.879	.879
326.2	24.76	.0890	26.56	23.22	.874	.879
350.1	26.56	.0931	28.21	24.93	.884	.874
376.3	28.55	.0944	30.58	26.79	.876	.873

District for the several reasons previously mentioned in the original paper. In addition, the desire was to measure the velocities as close to the wall as possible, and a very small diameter pitot tube seemed the best practical answer. By extending the velocity observations to a very short distance from the wall, the distance the velocity curve must be projected to meet the wall is minimized. Measurements as close as 0.15 inches from the wall invariably conformed to the velocity pattern in the balance of the pipe, as determined from a logarithmic plotting of velocity versus distance from the wall. Values at lesser distances were smaller than the plotting extension would indicate.

The installation of a fixed "standard" pitot tube at the center of the pipe as a metering or control device is unsuitable for the existing penstocks, since they are required to discharge debris of considerable size, up to six inches in diameter at times. Any fixed device projecting into the flow would be subject to considerable battering from such debris. The venturi meters used escape such treatment.

No influence of the supports on the pitot tube readings could be discerned from a comparison of the velocities taken on the leading support as against those on the trailing support, with the exception of trailing support readings taken one-quarter inch from the leading support. Readings in such close proximity to the leading support showed a very small reduction of velocity from the readings taken from the leading support.

The writer is indebted to Professor Steponas Kolupaila for his detailed analysis of the velocity distribution for the test data submitted to him. It should be mentioned that the recorded Q of 435 c.f.s. was merely a label given to this particular set of measurements for identification, and was not intended to accurately represent the true flow rate.

Since the velocity distributions obtained in the balance of the flow tests may be of interest, the test data for the other fourteen tests were plotted on log-log paper and straight lines fitted to each set of data by eye. The following results were obtained for the basic formula $V = ay^x$:

Tabular Material

Q Pitot	a	x	V Max.	V Mean	V Mean/V Max.	k
96.24	7.29	.0948	7.88	6.85	.742	.872
114.8	8.71	.0976	9.40	8.17	.869	.869
124.9	9.49	.0965	10.26	8.89	.867	.870
142.3	11.33	.0937	12.15	10.55	.868	.873
161.4	12.25	.0956	13.24	11.49	.868	.871
165.9	12.70	.0960	13.61	11.81	.868	.871
198.8	15.20	.0998	16.35	14.15	.865	.866
238.3	18.11	.0980	19.42	16.96	.873	.868
252.9	19.20	.0924	20.52	18.01	.878	.875
278.5	21.31	.0910	22.63	19.83	.876	.877
298.3	22.64	.0889	24.17	21.24	.879	.879
326.2	24.76	.0890	26.56	23.22	.874	.879
350.1	26.56	.0931	28.21	24.93	.884	.874
376.3	28.55	.0944	30.58	26.79	.876	.873

It will be noted that the values of k, which are computed from $\frac{2}{(x+1)(x+2)}$ correspond very closely to the ratio $\frac{V\ mean}{V\ max.}$, as noted by

Professor Kolupaila.

The writer is likewise indebted to Mr. R. C. Youngquist, A.M. ASCE, for his analysis of the flow data for the two pipes in terms of the Hazen-Williams pipe formula. The data were analyzed and presented according to what the writer believed were the two best known flow formulae, the Mannings and the Darcy. Other exponential type formulae, of which there are several, have been generally regarded with distrust by the writer because of their lack of dimensional homogeneity. However, the primary reason for submitting the data for publication was to make them available to the profession for its information and use. If the data are accepted and use can be made of them, then the writer's purpose has been accomplished.

CORRECTIONS TO SEPARATE NO. 297.—Page 297-3: In the equation in the middle of the page, the first term under the radical sign should be A_1^2.

Page 297-10: In the tenth line from the bottom of the page, "was" should be changed to "has". In the third line from the bottom of the page "* * *at least the higher* * *" should be "* * *at least at the higher* * *".

Page 297-11: In the third line, the division sign should be changed to a plus sign. In the fifteenth line, the symbol m should be deleted from the left-hand member of the Nikuradse equation, and the division sign in the righthand member changed to a plus sign. In the lower half of the page, the symbol alpha should be changed to delta (δ) in the five places where it appears. In the third full paragraph from the bottom of the page, the expression T_o/ρ should be changed to $\sqrt{T_o/\rho}$, and nu, where it occurs in the denominator of the expression representing alpha, should be changed to V.

Page 297-15: In the first paragraph, the two expressions should be:

von Karman: $$\frac{Vm-V}{V_*} = \frac{-1}{K} \left[\ln\left(1 - \sqrt{1-\frac{y}{r_o}}\right) + \sqrt{1-\frac{y}{r_o}}\right]$$

Prandtl: $$\frac{Vm-V}{V_*} = \frac{1}{K} \ln \frac{r_o}{y}$$

In the second paragraph, the third line should begin with the symbol V_*.

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a. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
b. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.
c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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